Math 1
Summer Assignment
2017

Assignment Due:
Monday, August 28, 2017

Name: ______________________________

The following packet contains topics and definitions that you will be required to know in order to succeed in Math 1 this year. You are advised to be familiar with each of the concepts and to complete the included problems. All of these topics are covered in 8th Grade Math and will be used frequently throughout the year. Each topic has a brief tutorial before the practice problems. If further instruction is needed, Kahn Academy and CK-12 are great resources.

Instructions:

- DO ALL PROBLEMS WITHOUT A CALCULATOR.

- Show all work, use another sheet of paper if needed. You may use your notes from previous mathematics courses to help you. You must do all work without any help from another person.

- ALL work should be completed and ready to turn in on the DUE DATE. ***There will be an assessment at the end of the second week of school on this material.***

- The packet will be worth 50 points for completion (YOU MUST SHOW YOUR WORK) and the packet test will be worth up to 100 points. Each day the packet is turned in late, you will lose 5 points.

- Spend a little bit of time each week to complete your packet.

- If you have any questions, you can reach me at paltman@thefsi.us. I will be checking my email periodically over the summer.

Section 1: Order of Operations

		P arenthesis
		E xponents
Which ever comes first, left to right.	{	**M** ultiplication
		D ivision
Which ever comes first, left to right.	{	**A** ddition
		S ubtraction

Example:

Simplify the following:

$\frac{(18+4)}{2} - 3(10 \cdot 2 - 3 \cdot 6)$ ←Work inside first set of parenthesis first

$= \frac{22}{2} - 3(10 \cdot 2 - 3 \cdot 6)$ ←Work inside second set of parenthesis by multiplying first

$= \frac{22}{2} - 3(20 - 18)$ ←Continue to work inside second set of parenthesis by subtracting

$= \frac{22}{2} - 3(2)$ ←Divide the fraction

$= 11 - 3(2)$ ←Multiply

$= 11 - 6$ ←Subtract

$= 5$

Simplify the following problems:

1) $4 + 6(8)$

2) $\frac{4(8-2)}{3+9}$

3) $4 \times 3^2 + 2$

4) $14 + 6 \times 2^3 - 8 \div 2^2$

5) $3 + 4[13 - 2(6 - 3)]$

6) $3(\frac{6+12}{2})$

7) $(3.4)(2.7) + 5$

8) $(6.88 \div 2) - (9.3 - 9.03)$

9) $(.9 + 1.1)^2 - (11^2 - 117)$

10) $\frac{2.4+3.5}{.7} \cdot 2$

11) $1\frac{2}{3} - \frac{3}{4} \cdot 4$

12) $\frac{3}{4} \div (\frac{1}{2})^2 + \frac{1}{2}$

Section 2: Real Number Comparison

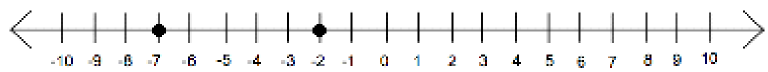
An Inequality is a mathematical sentence that compares the value of two expressions using an inequality symbol.

Inequality Symbol	Pronounced	Example
$<$	Less than	$4 < 9$
$\leq$	Less than or equal to	$-3 \leq 2$
$>$	Greater than	$-4 > -7$
$\geq$	Greater than or equal to	$5 \geq 5$
$\neq$	Not equal to	$7 \neq 11$

When comparing two numbers with an inequality symbol, it can be useful to plot both numbers on a number line. By plotting both numbers on a number line, you can see which number is greater simply by seeing which number is further to the right.

Example: Fill in the blank with the correct inequality symbol ($<$, $>$)

$$-7\_\_\_ -2$$

	**First plot a point on each number on a number line**
$-7 < -2$ ←-------------------------------	**Since -2 is further to the right, it is the larger number, therefore you use a less than sign because -7 is less than -2**

HINT: When comparing fractions you can either get a common denominator to compare, or convert the fraction(s) to decimals then compare.

Example:

Which is greater, $\frac{4}{9}$ or $\frac{5}{12}$?

Multiples of 9: 9, 18, 27, 36
Multiples of 12: 12, 24, 36

List multiples of each denominator to find their LCD.

Since the LCM of 9 and 12 is 36, the LCD of the fractions is 36.

$\frac{4}{9} = \frac{4 \cdot 4}{9 \cdot 4}$ ← Multiply the numerator and denominator by 4.

$= \frac{16}{36}$ ← Simplify.

$\frac{5}{12} = \frac{5 \cdot 3}{12 \cdot 3}$ ← Multiply the numerator and denominator by 3.

$= \frac{15}{36}$ ← Simplify.

Since $\frac{16}{36} > \frac{15}{36}$,

$\frac{4}{9} > \frac{5}{12}$

Use $<$, $=$, $>$ to compare the following sets of numbers:

13. 2 ___ 6 **14.** − 12 ___ − 15 **15.** 2 ___ − 1

16. 0.63 ___ 0.6 **17.** $\frac{8}{9}$ ___ 0.88 **18.** -1.45 ___ 1.45

19. $\frac{2}{3}$ ___ $\frac{1}{6}$ **20.** $\frac{3}{4}$ ___ $\frac{12}{16}$ **21.** $-2\frac{5}{8}$ ___ $-2\frac{1}{2}$

Section 3: Variables and Verbal Expressions

Write each phrase as an algebraic expression.

Phrase	**Expression**
nine increased by a number x	9 + x
fourteen decreased by a number p	14 - p
seven less than a number t	t - 7
the product of 9 and a number n	9 • n or 9n
thirty-two divided by a number y	32 ÷ y or $\frac{32}{y}$

Write an algebraic expression for each phrase.

22. 7 increased by *x* **23.** *p* multiplied by 3 **24.** 10 decreased by *m*

25. *n* less than 7 **26.** the product of 2 and *q* **27.** 3 more than *m*

28. the difference of 8 and a number **29.** the sum of 4 and a number

30. the product of 2 and a number **31.** 3 increased by a number

32. 10 plus the quotient of a number and 15 **33.** 12 less than a number

Section 4: Evaluating Algebraic Expressions

A **variable** is a letter, for example x, y or z, that represents an unspecified number.

To evaluate an algebraic expression, you have to substitute a number for each variable and perform the arithmetic operations.

Example: Calculate the following expression for x = 3 and z = 2

$$6z + 4x = ?$$

Solution: Replace x with 3 and z with 2 to evaluate the expression. (Be sure to use parenthesis when you substitute!)

$$6z + 4x = ?$$
$$6(2) + 4(3) = ?$$
$$12 + 12 = 24$$

Evaluate each expression for the given values.

34. $xy \quad for\ x = 3\ and\ y = 5$

35. $2 + n \quad for\ n = 3$

36. $10 - r + 5 \quad for\ r = 9$

37. $m + n \div 6 \quad for\ m = 12\ and\ n = 18$

38. $4m + 3 \quad for\ m = 5$

39. $35 - 3x \quad for\ x = 10$

40. $3ab - c \quad for\ a = 4,\ b = 2,\ c = 5$

41. $\frac{ab}{2} + 4c \quad for\ a = 6, b = 5, c = 3$

Section 5: Solving One- Step Equations

A **one-step equation** is as straightforward as it sounds. You will only need to perform one step in order to solve the equation. The goal in solving an equation is to only have a variable on one side of the equal sign and numbers on the other side of the equal sign.

The strategy for getting the variable by itself involves using opposite operations. The most important thing to remember in solving a linear equation is that whatever you do to one side of the equation, you MUST do to the other side. So if you subtract a number from one side, you MUST subtract the same value from the other side. You will see how this works in the examples.

Example:

Solve $-2 = k - 14$.

$-2 = k - 14$ ← Since 14 is subtracted from k, you must add 14 to each side to isolate k.

$-2 + 14 = k - 14 + 14$ ← Add 14 to each side.

$12 = k$ ← Simplify.

Example:

Solve $\frac{x}{-7} = 15$.

$\frac{x}{-7} = 15$ ← Since x is divided by -7, you must multiply each side by -7 to isolate x.

$-7 \cdot \left(\frac{x}{-7}\right) = -7 \cdot 15$ ← Multiply each side by -7.

$x = -105$ ← Simplify.

Example:

Solve $816 = 8c$.

$816 = 8c$ ← Since c is multiplied by 8, you must divide each side by 8 to isolate c.

$\frac{816}{8} = \frac{8c}{8}$ ← Divide each side by 8.

$102 = c$ ← Simplify.

Solve the following one-step equations:

42. $x - 2 = 6$

43. $y + 1.5 = 3.7$

44. $2a = 22$

45. $\frac{3}{4}x = 12$

46. $\frac{1}{4}x = \frac{5}{8}$

47. $\frac{x}{3} = 3$

Section 6: Solving Two-Step Equations

When solving a two-step equation, you will need to perform two steps in order to solve the equation.

The goal in solving a two step equation is the same as in solving a one step: to only have a variable on one side of the equal sign and numbers on the other side of the equal sign.

The strategy for getting the variable by itself with a coefficient of 1 involves using opposite operations. The most important thing to remember in solving a linear equation is that whatever you do to one side of the equation, you MUST do to the other side. So if you subtract a number from one side, you MUST subtract the same value from the other side. You will see how this works in the examples.

In solving two-step equations you will make use of the same techniques used in solving one-step equation only you will perform two operations rather than just one. (Note: you should always add or subtract first, then multiply or divide)

Example:

Solve $\frac{n}{5} - 7 = -9.$

$\frac{n}{5} - 7 + 7 = -9 + 7$ ← **Add 7 to each side.**

$\frac{n}{5} = -2$ ← **Simplify.**

$(5)\frac{n}{5} = (5)(-2)$ ← **Multiply each side by 5.**

$n = -10$ ← **Simplify.**

Example:

Solve $125 + 3b = 154.97$

$125 + 3b = 154.97$

$125 - 125 + 3b = 154.97 - 125$ ← **Subtract 125 from each side.**

$3b = 29.97$ ← **Simplify.**

$\frac{3b}{3} = \frac{29.97}{3}$ ← **Divide each side by 3.**

$b = 9.99$ ← **Simplify.**

Solve the following two-step equations:

48. $1 + \frac{a}{5} = -1$

49. $-1 = 3 + 4x$

50. $\frac{x}{3} - 9 = 0$

51. $\frac{5}{7}x + \frac{1}{7} = 3$

52. $\frac{1}{4}x - \frac{3}{4} = \frac{5}{8}$

53. $0.4x + 9.2 = 10$

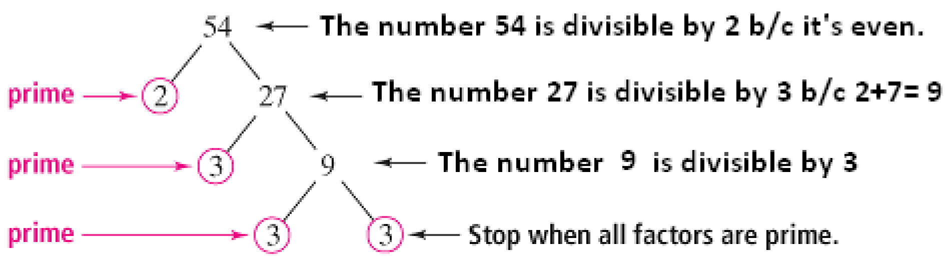
Section 7: Divisibility and Factors

A number is **divisible** by a second number if the number can be divided by the second number with a remainder of 0.

Divisible By:	Divisibility Test:
2	The number is even
3	The sum of the number's digits is divisible by 3
4	The last two digits are divisible by 4 or ends in 00
5	The number ends in a 5 or 0
6	The number is divisible by 2 and 3
8	The last three digits are divisible by 8 or ends in 000
9	The sum of the number's digits is divisible by 9
10	The number ends in a 0

A composite number written as a product of prime numbers is the prime factorization of the number. You can use the divisibility tests and a factor tree to find the prime factorization of a number.

Example #1: Use a factor tree to find the prime factorization of 54.



So, the prime factorization of 54 is $2 \bullet 3 \bullet 3 \bullet 3 = 2 \bullet 3^3$.

Use the divisibility tests to determine whether the first number is divisible by the second?

54. 325 by 5 **55.** 790 by 2 **56.** 450 by 9 **57.** 364 by 4

Identify each number as *prime or composite*. If the number is composite, find its prime factorization by making a factor tree.

58. 30 **59.** 71 **60.** 61 **61.** 37 **62.** 38

Section 8: Measures of Central Tendency

In working with statistical data, it is often useful to determine a single quantity that best describes the set of data. The best quantity to choose is usually one of the most popular measures of central tendency: the mean, the median, or the mode.

Definitions:

Mean	The **mean** is the sum of the data items in a set divided by the number of data items in the set.
Median	The **median** is the middle value in a set of data when the numbers are arranged in numerical order. If the set has an even number of data items, the median is the mean of the two middle data values.
Mode	The **mode** is the data item that occurs most often in a data set.

Example:

Find the mean, median, and mode of the set of data: 34 46 31 40 33 40.

Mean: $\frac{34+46+31+40+33+40}{6}$ **←Add the data items and ←divide by the number of data items in the set**

$= \frac{224}{6} = 37.\overline{3}$

Median: 31 33 34 40 40 46 **←Arrange the data items in increasing order.**

$\frac{34+40}{2} = \frac{74}{2} = 37$ **←Since there is an even number of data values, find the mean of the two middle data values.**

Mode: = 40 **←The mode is 40 since it occurs most often.**

Find the mean, median, and mode of each set of data.

63. daily sales of a store: \$834 \$1099 \$775 \$900 \$970

64. number of points scored in 8 soccer games: 0 10 4 11 7 6 3 2

65. number of days above 50°F in the last five months: 6 8 15 22 9

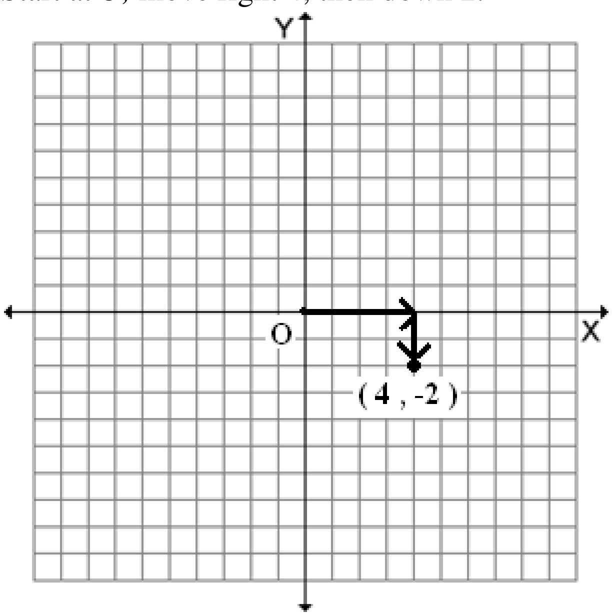
66. heights of players on a basketball team in inches: 72 74 70 77 76 72

Section 9: Plotting on the Coordinate Plane

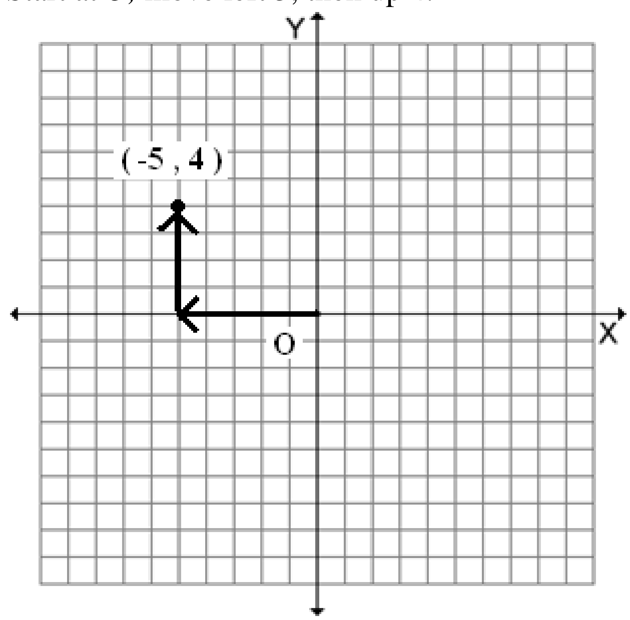
You can graph a point on a *coordinate plane.* Use an *ordered pair* (*x*, *y*) to record the coordinates. The first number in the pair is the *x-coordinate.* The second number is the *y-coordinate.*

To graph a point, start at the origin, *O*. Move horizontally according to the value of *x*. Move vertically according to the value of *y*.

Example 1: Plot the ordered pair (4, -2)
Start at *O,* move right 4, then down 2.



Example 2*:* Plot the ordered pair (-5, 4)
Start at *O,* move left 5, then up 4.



List the ordered pair for each letter, then name the Quadrant the point lies in.

67. P = (,) Quadrant _______

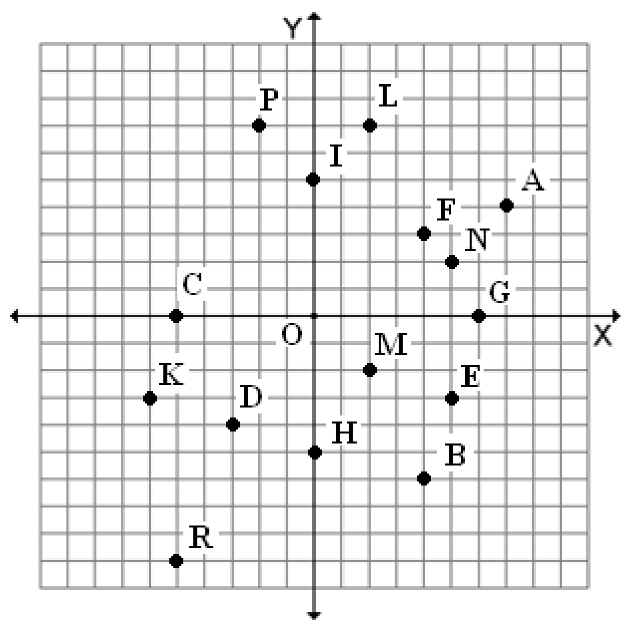
68. B = (,) Quadrant _______

69. K = (,) Quadrant _______

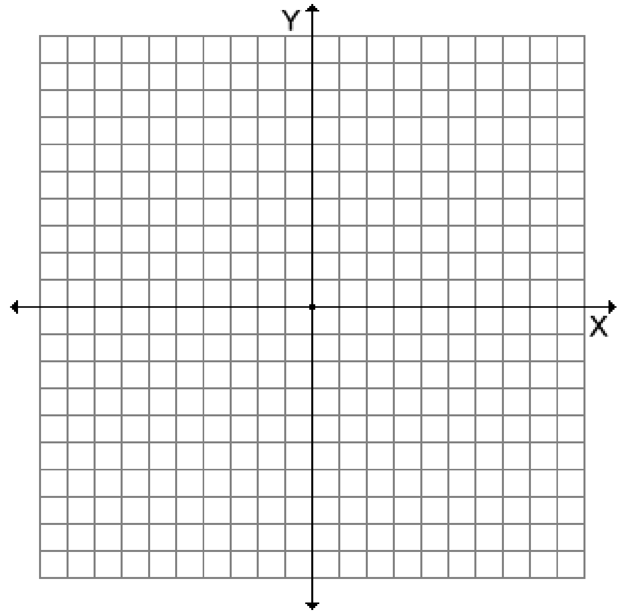
70. A = (,) Quadrant _______

71. F = (,) Quadrant _______

72. D = (,) Quadrant _______



Plot the following ordered pairs on the coordinate plane at right and label with the corresponding letter:



73. H = (-2 , 2)

74. W = (0 , 0)

75 S = (-2, -2)

76. J = (2 , 2)

77. P = (-1 , -3)

78. C = (1 , -3)

79. V = (2 , -2)

Section 10: Fractions

Simplify:

80. $\frac{8}{24}$

81. $\frac{21}{14}$

82. $\frac{5}{20}$

Write the following mixed numbers as improper fractions:

83. $2\frac{1}{7}$

84. $-5\frac{7}{8}$

85. $6\frac{3}{7}$

Perform the indicated operation, and simplify if possible:

86. $\frac{5}{4}+\frac{3}{4}$

87. $\frac{6}{7}+\frac{3}{2}$

88. $\frac{9}{2}+\frac{7}{5}$

89. $\frac{7}{8}-\frac{1}{2}$

90. $\frac{15}{8}-\frac{12}{5}$

91. $-\frac{3}{5}-\frac{2}{7}$

92. $\frac{2}{3}*\frac{5}{8}$

93. $-\frac{5}{3}*\frac{2}{5}$

94. $\frac{4}{7}*\frac{8}{3}$

95. $\frac{1}{3}\div\frac{5}{2}$

96. $\frac{1}{9}\div\frac{7}{8}$

97. $-\frac{4}{5}\div\frac{1}{6}$

98. $6*\frac{4}{5}$

99. $15\div\frac{3}{8}$

100. $\frac{2}{7}*14$

<u>**Section 11: Solving Inequalities**</u>

⌧ *Explanation*

An inequality is a math sentence with one of the following inequality signs: $<, >, \leq,$ or $\geq$.

Because inequalities have a range of values that represent their solution, we use a graph to represent those values. Just like equations, solving an inequality begins with isolating the variable using inverse operations. Once the variable is isolated you will use the chart below to graph a circle and line.

↓ Inequality ↓	**Circle**	**Line**
$x < \ldots$	empty ○	points left ←
$x > \ldots$	empty ○	points right →
$x \leq \ldots$	filled ●	points left ←
$x \geq \ldots$	filled ●	points right →

Some unique properties about inequalities follow:

~ The inequality sign will change direction from < to > or from ≤ to ≥ if while you are solving the following occurs:

1] you multiply or divide…
2] on both sides of the inequality…
3] by a negative number.

~Depending on where the variable is, a single inequality sign can be read two different ways:

$x < 3$ is read as, "x is less than three," OR "three is greater than x."
$5 \geq y$ is read as, "five is greater than or equal to y," OR "y is less than or equal to five."

⌧ *Examples*

Solve and graph the following: $-2x + 4 < 14$

$-2x + 4 - 4 < 14 - 4$ — Subtract 4 from both sides to begin isolating x.

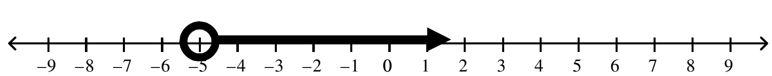
$-2x < 10$ — Divide by -2 on both sides to finish isolating x.

$x > -5$ — If you DIVIDE on BOTH sides by a NEGATIVE number, the inequality sign must change direction (see E*xplanation*).

$x > -5$ — The variable is isolated and ready to be graphed.
The inequality is read; "x is greater than negative five."

The circle will be on -5.
The circle will be empty because the sign is not "or equal to."

The arrow will be to the right because the sign is "greater than."



⌦*Problems*

Solve and graph the following inequalities. You must draw your own number line.

13. $3x+5>17$

14. $6\geq 4+\frac{x}{2}$

15. $\frac{x+6}{-3}<2$

16. $-3(2x+4)\leq 0$

IV. Solving Absolute Value Equations and Inequalitities

⌦*Explanation*

Absolute value is defined as a number's distance from zero on a number line.

Distance is defined such that it is always positive: $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Because distance is always positive, Absolute Value always PRODUCES positive numbers. However, its inputs can be any number at all. Consider the following examples:

The $|5|$ is equal to 5.

The $|-5|$ is also equal to 5.

Although the output is the same in both examples, the inputs are different. Because we are looking for inputs when we solve an absolute value equation, there will inevitably be two solutions. So to solve an absolute value equation, you must isolate the absolute value expression, and then split the equation into two possible solution paths; one positive and one negative.

⌧ *Examples*

Solve: $2|x+3|-1=9$

$2|x+3|-1=9$ Add 1 to both sides to being isolating absolute value expression in the equation.

$2|x+3|=10$ Divide both sides by 2 to fully isolate the absolute value expression.

$|x+3|=5$ The absolute value expression is isolated, so split the equation into a positive path and a negative path. This will generate two solutions.

$x+3=5$ $\quad x+3=-5$ Solve both equations independently by subtracting three on both sides.

$x=2$ $\quad x=-8$ The two solutions that would make the equation true are 2 and -8.

⌧*Problems*

Solve the following equations.

17. $3|x+5|=21$

18. $19=3+4|x-1|$

19. $\frac{|2x+3|}{5}=-11$

20. $27=4\left|\frac{x}{-2}+5\right|-9$

⌧*Explanation*

An Absolute Value Inequality is a math sentence with an absolute value expression and an inequality sign. To solve an absolute value inequality, you will still isolate the absolute value expression first. Once Isolated, you will create two inequalities to solve, and then you will graph both solutions on one number line.

1. Set up two inequalities like this (without the absolute value symbols):

$$(\text{absolute value expression}) > (\text{other side})$$

$$(\text{absolute value expression}) < -(\text{other side})$$

2. Solve each inequality and graph
3. Complete the solution as an 'and' or an 'or'

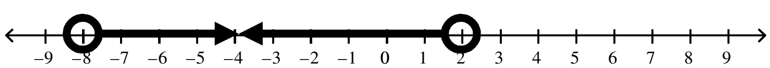
⌧ *Examples*

Solve: $2|x+3|<10$

$|x+3|<5$ *Divide by 2 on both sides to isolate the absolute value expression.

$x+3<5$ $x+3>-5$ *Split the equation into two paths using the setup given above.

$x<2$ $x>-8$ *Graph both solutions on one number line.



$-8<x<2$ *Write your solution

⌧*Problems*

Solve and graph the following inequalities.

21. $3|x+5|<21$

22. $|x-1|-4>14$

23. $1+5\left|\frac{x}{2}\right|\leq 6$

24. $-8\leq -4|x-2|$